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On the Reduction of Transit Observations by the Method of Least Squares. By Harold Jacoby, B.A.

When it is desired to adjust a series of observations by the method of least squares, the process usually employed is not always the most advantageous one available—in fact, the usual method is really an ingenious way of finding the *adjusted* values of the unknowns from the coefficients and absolute terms of the original observation equations. Occasionally, however, these coefficients are themselves all functions of some single quantity. In such a case there must exist a functional relation between the adjusted values of the unknowns and the single quantity in question; and this relation may possibly be evaluated *without computing the actual coefficients of the observation equations.* The above applies to the transit instrument in the meridian, for the coefficients of the observation equations, as well as the weight-factor, are then all functions of the declination. The *latitude* affects them as a constant only.

Before going into the details of the application of the above remarks to actual work, it is desirable to call attention to a theorem in the method of least squares which is very often of considerable use. Let there be given two groups of linear observation equations of the form:

$$
\begin{array}{ll}
a_1 w + b_1 y + c_1 z + d_1 = 0, & a_1' x + b_1' y + c_1' z + d_1' = 0, \\
a_2 w + b_2 y + c_2 z + d_2 = 0, & a'_2 x + b_2' y + c_2' z + d_2' = 0, \\
\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot
\end{array}
$$

in which it will be seen that the equations of the first group lack the term in x, and those of the second group lack the term in w. Now let normal equations be formed from each group as follows:

$$
\begin{array}{rr}
[aa]w + [ab]y + [ac]z + [ad] = 0, & [a'a']x + [a'b']y + [a'c']z + [a'd'] = 0. \\
[bb]y + [bc]z + [bd] = 0, & [b'b']y + [b'c']z + [b'd'] = 0. \\
[cc]z + [cd] = 0, & [c'c']z + [c'd'] = 0, \\
[dd] & [d'd'].
\end{array}
$$

Let w and x be eliminated in the usual way, giving:

$$
\begin{array}{rr}
[bb\,.\,1]y + [bc\,.\,1]z + [bd\,.\,1] = 0, & [b'b'\,.\,1]y + [b'c'\,.\,1]z + [b'd'\,.\,1] = 0, \\
[cc\,.\,1]z + [cd\,.\,1] = 0, & [c'c'\,.\,1]z + [c'd'\,.\,1] = 0, \\
[dd\,.\,1] & [d'd'\,.\,1].
\end{array}
$$

Now let these equations be added, term by term, giving:

$$
\begin{array}{r}
([bb\,.\,1] + [b'b'\,.\,1])y + ([bc\,.\,1] + [b'c'\,.\,1])z + ([bd\,.\,1] + [b'd'\,.\,1]) = 0, \\
([cc\,.\,1] + [c'c'\,.\,1])z + ([cd\,.\,1] + [c'd'\,.\,1]) = 0, \\
([dd\,.\,1] + [d'd'\,.\,1]);
\end{array}
$$

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then the equations last obtained are *identically* the same as would have resulted if we had formed a set of normal equations from the two original groups of observation equations, taken altogether as a single group, and then eliminated w and x according to the usual rule.*

When transit observations are reduced upon the assumption that the azimuth changed when the instrument was reversed,† we have two groups of observation equations of the kind just mentioned, and may take advantage of the above principle. It is, perhaps, out of place here to remark upon the desirability of making such an assumption, since the method to be given is equally applicable in any case. Perhaps the strongest argument against the use of a least square solution, upon the hypothesis of two values of the azimuth, is that the actual observation times of the reversal are in no way affected by the azimuth of the instrument before reversal, and ought therefore to have no influence in the determination of that quantity. It is certain, however, that they do obtain such an influence, if a least square adjustment is made upon the assumption of two azimuths. Yet the same objection will apply to observations of any kind in which some of the observation equations have one or more zero coefficients, a case occurring very frequently in practice. We have abundant authority for applying the method of least squares in such cases. It only remains to inquire whether the hypothesis of a change of azimuth is warranted by facts. Does such a change really occur? Upon this point no decisive evidence has been published, so far as I know. It must, therefore, be left to each observer to decide whether he has sufficient skill to reverse his instrument with a certainty of not appreciably changing the azimuth adjustment. If so, he would violate a known fact, by assuming that the azimuth changed in reversing. It seems, however, that in the majority of cases it is quite safe to assume two azimuths. It will be shown below that it is possible to make separate reductions on *both* hypotheses without very extensive extra calculations.

So much being premised, we can proceed to explain the proposed method of reduction, beginning with the process applicable to the hypothesis of *no change of azimuth.* If we assume the usual notation, we shall have, by Mayer's formula, for each observation a weighted equation of the form:

$$\sqrt{p}\,\mathrm{A}x \pm \sqrt{p}\,\mathrm{C}y + \sqrt{p}\,\mathrm{B}z + \sqrt{p}\,l = 0 \begin{cases} \text{Clamp West.} \\ \text{Clamp East.} \end{cases}$$

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* For a demonstration of this interesting theorem, see *Jordan, Handbuch der Vermessungskunde, erster Band* (*Ausgleichungsrechnung*) *dritte Auflage,* 1888, p. 78.

† Chauvenet, *Spherical and Practical Astronomy,* vol. ii., p. 201. Doolittle, *Practical Astronomy,* p. 325.

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in which, as usual:

$$A = \frac{\sin(\phi - \delta)}{\cos\delta}, \quad B = \frac{\cos(\phi - \delta)}{\cos\delta}, \quad C = \frac{1}{\cos\delta}, \quad p = \text{weight},$$

$$l = T - a + \Delta\theta_0 + Aa_0 + Bb \pm Cc_0 - 0^s.021 \cos\phi \sec\delta \begin{cases} \text{Clamp West}, \\ \text{Clamp East}, \end{cases}$$

and $\Delta\theta_0$, a_0, and c_0 are approximate values of the chronometer correction, azimuth constant, and collimation constant.* After the solution has been carried out, we have as the adjusted values—

$$a = a_0 + x, \quad c = c_0 + y, \quad \Delta\theta = \Delta\theta_0 + z.$$

Now let us write—

$$m_1 = \pm p \sec\delta, \; m_2 = \pm p \tan\delta \sec\delta, \; m_3 = p, \; m_4 = p \tan\delta, \; m_5 = p \tan^2\delta,$$

$$m_6 = p \sec^2\delta \begin{cases} \text{Clamp West}, \\ \text{Clamp East}, \end{cases}$$

and put

$$\begin{aligned} B' &= \sin\phi\,[m_1] - \cos\phi\,[m_2], \\ C' &= \sin\phi\,[m_3] - \cos\phi\,[m_4], \\ L &= \sin\phi\,[m_3 l] - \cos\phi\,[m_4 l], \\ K &= \sin^2\phi\,[m_3] + \cos^2\phi\,[m_5] - 2\sin\phi\cos\phi\,[m_4]. \end{aligned}$$

We do not then need to form the usual normal equations, which would be

$$\begin{aligned} [pAA]z + [pAC]y + [pA]z + [pAl] &= 0, \\ [pCC]y + [pC]z + [pCl] &= 0, \\ [p]z + [pl] &= 0, \\ [pll]&, \end{aligned} \qquad (a)$$

for we can write at once the *reduced* normal equations

$$\begin{aligned} [pCC\,.\,1]y + [pC\,.\,1]z + [pCC\,.\,1] &= 0, \\ [p\,.\,1]z + [pl\,.\,1] &= 0, \\ [pll\,.\,1]&, \end{aligned} \qquad (b)$$

by means of the relations

$$[pCC\,.\,1] = [m_6] - \frac{B'^2}{K}, \quad [pC\,.\,1] = [m_1] - \frac{B'C'}{K}, \quad [pCl\,.\,1] = [m_1 l] - \frac{B'L}{K},$$

$$[p\,.\,1] = [m_3] - \frac{C'^2}{K}, \quad [pl\,.\,1] = [m_3 l] - \frac{C'L}{K},$$

$$[pll\,.\,1] = [m_3 ll] - \frac{L^2}{K}.$$

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* In this investigation, the collimation constant is considered positive when stars at upper transit require a positive correction for clamp West. It should also be noticed that when the instrument is *very* well adjusted, a_0 and c_0 can be put equal to 0, whereby the computation of l is much facilitated.

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From the equations (*b*), now involving only two unknowns, we get the adjusted values y and z in the usual way. If we wish the value of x also, we get it from the equation—

$$x = -\frac{B'}{K}y - \frac{C'}{K}z - \frac{L}{K}, \qquad (c)$$

where all the coefficients in the second member are known. It will be noticed that the quantities B′, C′, L, K are really coefficients of the usual normal equations (*a*), but by proceeding at once to the equations (*b*), we point out the method of reduction for the case of *two* different azimuth constants. We have then, in accordance with the principle already explained, to proceed by the following rule:—

Carry out the reduction for the clamp West and clamp East observations separately, until two separate sets of equations of the form (*b*) are obtained. Add corresponding coefficients in the two sets, and solve the resulting set in the usual way.

The values of y and z having thus been obtained, the two values of x for clamp West and clamp East result from an application of (*c*) to the clamp West and clamp East observations respectively. It is, of course, obvious that separate values of a_0 may be assumed in the beginning for the clamp West and clamp East observations, and separate values of B′, C′, L, K will also be found. The peculiar advantage of this method arises from the possibility of tabulating the quantities m_1, m_2, m_3, m_4, m_5, m_6, in tables of single entry. As they are functions of δ only, the values given in the tables which I have computed are good for any latitude. In computing the tables Safford's weight formula* was used. This gives

$$p = \frac{1{\cdot}3}{1 + 0{\cdot}3 \sec^2\delta},$$

which is often intermediate between the values given by other well-known formulas.† After all, we need only an approximate value of the weight; and, if necessary, tables to agree with any other weight formula could easily be computed from mine by simply multiplying each number by the ratio of the two weights at that particular declination.

It is evident that after an adjustment has been made upon the hypothesis of two azimuths, it is quite easy to obtain the values that would have resulted from the hypothesis of an unchanged azimuth; and such a double solution applied to a number of cases would throw much light upon the question of the desirability of the assumption of two azimuths.

[The tables referred to have been communicated by the author in manuscript. They are preserved in the archives of the Society.]

* *Transactions of the Wisconsin Academy of Science*, vol. vii., 1883–87, p. 199.

† Young, *Washington Observations*, 1878, Appendix ii., p. 29.

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Observation of Comet b 1891 (*Wolf*), *made at the Royal Observatory, Greenwich.*

(*Communicated by the Astronomer-Royal.*)

The observation was made with the East or Sheepshanks Equatoreal, aperture 6·7 inches, by taking transits over two cross-wires at right angles to each other, and each inclined 45° to the parallel of declination. Magnifying power 55.

Greenwich Mean Solar Time.	Observer.	☄ − ✱ R.A.	Corr. for Parallax.	Corr. for Refraction.	☄ − ✱ N.P.D.	Corr. for Parallax.	Corr. for Ref.	No. of Comps.	Apparent R.A.	Tabular R.A.	Apparent N.P.D.	Tabular N.P.D.
d h m s		s	s	s	′ ″	″	″		h m s	h m s	° ′ ″	° ′
Dec. 2 10 10 41	A.C.	−29·90	−·15	+·05	+15 6·1	−8·5	+2·7	5	4 23 36·3	4 23 37	103 32 9·7	103 32·0

Assumed Mean Place of Comparison Star.

Name and Authority.	R.A. 1891·0.	N.P.D. 1891·0.
	h m s	° ′ ″
Greenwich 10-Year Catalogue 1880, No. 718	4 24 3·15	103 17 21·1

In computing the parallax log Δ has been assumed 9·9703, this being interpolated from Berberich's Ephemeris, as is also the Tabular Place. Here and in the observations communicated in the *Monthly Notices* for November, a second term has been applied in the N.P.D. Corr. for Refraction, depending on the difference between apparent and true orientation of the wires; this point was discussed by Col. Tupman in vol. xlviii., p. 96, of the *Monthly Notices.*

The initials "A.C." are those of Mr. Crommelin.